

84 Product Operator Treatment

as a "refocusing routine" and this brings us directly from $D(1)$ to $D(4)$.

$$\begin{aligned} D(1) &\xrightarrow{180_x A} +(p'/n)\{z1\} + q'\{1y\} \\ &\xrightarrow{t_e(\text{shiftX})} (p'/n)\{z1\} + q'\cos\Omega_x t_e\{1y\} - q'\sin\Omega_x t_e\{1x\} \end{aligned} \quad (\text{II.42})$$

D(4)

With the assumption that $\Delta_1 = 1/2J$ (i.e., $\pi J\Delta_1 = \pi/2$) we have

$$\begin{aligned} D(4) &\xrightarrow{\Delta_1(J)} (p'/n)\{z1\} - q'\cos\Omega_x t_e\{zx\} - q'\sin\Omega_x t_e\{zy\} \\ &\xrightarrow{\Delta_1(\text{shiftX})} (p'/n)\{z1\} - q'\cos\Omega_x t_e(c'\{zx\} + s'\{zy\}) \\ &\quad - q'\sin\Omega_x t_e(c'\{zy\} - s'\{zx\}) \end{aligned} \quad (\text{II.43})$$

D(5)

where c', s' have the same meaning as in (II.29), i.e.,

$$c' = \cos\Omega_x \Delta_1 \quad ; \quad s' = \sin\Omega_x \Delta_1$$

Using again the trigonometric relations for the sum of two angles as we did in (II.30) we rewrite $D(5)$ as:

$$D(5) = (p'/n)\{z1\} - q'\{zx\}\cos\Omega_x(t_e + \Delta_1) - q'\{zy\}\sin\Omega_x(t_e + \Delta_1) \quad (\text{II.44})$$

$$D(5) \xrightarrow{90_x X} (p'/n)\{z1\} - q'\{zx\}\cos\Omega_x(t_e + \Delta_1) - q'\{zz\}\sin\Omega_x(t_e + \Delta_1)$$

D(6)

$$D(6) \xrightarrow{90_x A} -(p'/n)\{y1\} + q'\{yx\}\cos\Omega_x(t_e + \Delta_1) + q'\{yz\}\sin\Omega_x(t_e + \Delta_1) \quad (\text{II.45})$$

D(7)

$$D(7) = -(p'/n)\{y1\} + q'\{yz\}\sin\Omega_x(t_e + \Delta_1) + \text{NOT} \quad (\text{II.46})$$

Up to this point we have followed step by step the calculations in Section II.9, while formally replacing $[]$ by $\{ \}$ and using p'/n instead of p' . This procedure is always valid for rotations (pulses) and in this case it was allowed for evolutions as stated in Appendix L, rule #4.

For the coupled evolution Δ_2 we can concentrate on the observable terms $\{x1\}, \{y1\}$ only and take advantage of the exception

stated in Appendix L, rule #5.

$$D(7) \xrightarrow{\Delta_2} -(p'/n)(-sC^n\{x1\} + cC^n\{y1\}) + q'\sin\Omega_X(t_e + \Delta_1)(-cSC^{n-1}\{x1\} - sSC^{n-1}\{y1\}) + \text{NOT} \quad (\text{II.47})$$

D(8)

n = number of magnetically equivalent X nuclei (e.g., number of protons in CH_n).

$$c = \cos\Omega_A\Delta_2 \quad C = \cos\pi J\Delta_2$$

$$s = \sin\Omega_A\Delta_2 \quad S = \sin\pi J\Delta_2$$

In order to separate the effect of shift (c and s) and coupling (C and S) we rewrite D(8) as

$$D(8) = -(p'/n)C^n(c\{y1\} - s\{x1\}) - q'\sin\Omega_X(t_e + \Delta_1)SC^{n-1}(c\{x1\} + s\{y1\}) \quad (\text{II.48})$$

Here again the second term is the true 2D signal while the first one generates axial peaks in the 2D picture since it does not contain the frequency Ω_X .

The value $\Delta_2 = 1/2J$, which implies $S = 1$ and $C = 0$, is not suitable anymore because it nulls the useful 2D signal for any $n > 1$. We will discuss later the optimum value of Δ_2 .

In writing the magnetization components we have to follow the procedure stated as rule #6 in Appendix L.

$$\begin{aligned} M_{xA}(8) &= - (nM_{oA}/p')(\text{coefficient of } \{x1\}) \\ &= -M_{oA}C^n \sin\Omega_A\Delta_2 + nM_{oA}(q'/p')SC^{n-1} \sin\Omega_X(t_e + \Delta_1) \cos\Omega_A\Delta_2 \\ M_{yA}(8) &= - (nM_{oA}/p')(\text{coefficient of } \{y1\}) \\ &= M_{oA}C^n \cos\Omega_A\Delta_2 + nM_{oA}(q'/p')SC^{n-1} \sin\Omega_X(t_e + \Delta_1) \sin\Omega_A\Delta_2 \\ M_{TA}(8) &= M_{xA}(8) + iM_{yA}(8) \\ &= iM_{oA}C^n \exp(i\Omega_A\Delta_2) + nM_{oA}(q'/p')SC^{n-1} \sin\Omega_X(t_e + \Delta_1) \exp(i\Omega_A\Delta_2) \\ &= M_{oA}[iC^n + n(q'/p')SC^{n-1} \sin\Omega_X(t_e + \Delta_1)] \exp(i\Omega_A\Delta_2) \quad (\text{II.49}) \end{aligned}$$

The decoupled evolution t_d during which the detection takes place is treated the same way it was done in II.9, as a rotation about Oz .

$$\begin{aligned} M_{TA}(9) &= M_{TA}(8) \exp(i\Omega_A t_d) \\ &= M_{oA} [iC^n + n(q'/p')SC^{n-1} \sin \Omega_X(t_e + \Delta_1)] \exp[i\Omega_A(t_d + \Delta_2)] \end{aligned} \quad (\text{II.50})$$

The enhancement factor of the 2D term is

$$n(q'/p')SC^{n-1}.$$

Now we can discuss the optimum value of Δ_2 . In Figure II.4 the value of the product nSC^{n-1} is plotted versus $\Delta_2 J$ for $n = 1, 2$ and 3 . The optimum Δ_2 values are:

$$\begin{aligned} n=1 \text{ (CH)} &\longrightarrow \Delta_2 = 0.5/J \\ n=2 \text{ (CH}_2\text{)} &\longrightarrow \Delta_2 = 0.25/J \\ n=3 \text{ (CH}_3\text{)} &\longrightarrow \Delta_2 = 0.196/J \end{aligned} \quad (\text{II.51})$$

A good compromise is $\Delta_2 = 0.3/J$ for which all three expressions S , $2SC$, and $3SC^2$ have values exceeding 0.8. The bad news is that this Δ_2 value does not cancel the axial peak, represented by the term iC^n in (II.50).

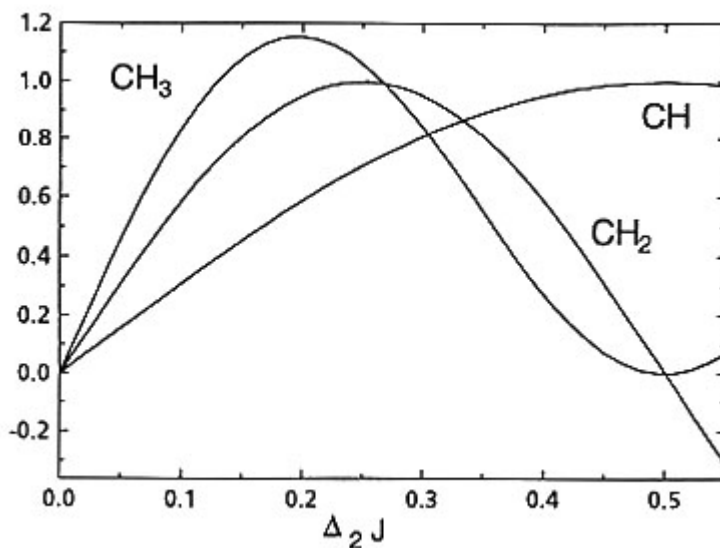


Figure II.4. Dependence of the factor nSC^{n-1} (S for CH, $2SC$ for CH_2 , and $3SC^2$ for CH_3) on $\Delta_2 J$.